

$$\begin{aligned}
 1a) \quad U_{n+1} &= \begin{pmatrix} u_{n+1} \\ u_{n+2} \end{pmatrix} = \begin{pmatrix} u_{n+1} \\ 3u_{n+1} - 2u_n \end{pmatrix} \\
 &= \begin{pmatrix} 0 & u_n + 1 & u_{n+1} \\ -2u_n + 3 & u_{n+1} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} u_n \\ u_{n+1} \end{pmatrix} \\
 &= A U_n.
 \end{aligned}$$

1b) Sachant $U_{n+1} = A U_n$, on obtient par récurrence que $\forall n \in \mathbb{N} \quad U_n = A^n U_0$.

$$2a) \quad P = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$$

Avec la calculatrice $P^{-1} = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$

$$\begin{aligned}
 \text{d'où} \quad P^{-1} A P &= \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \\
 &= \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ -1 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}
 \end{aligned}$$

$$2b) \text{ Sachant } P^{-1}AP = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

$$\text{ona : } (P^{-1}AP)^n = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}^n$$

$$\underbrace{(P^{-1}AP) \times \dots \times (P^{-1}AP)}_{n \text{ fois}} = \begin{pmatrix} 1^n & 0 \\ 0 & 2^n \end{pmatrix}$$

$$P^{-1}A^n P = \begin{pmatrix} 1 & 0 \\ 0 & 2^n \end{pmatrix}$$

$$P P^{-1} A^n P P^{-1} = P \begin{pmatrix} 1 & 0 \\ 0 & 2^n \end{pmatrix} P^{-1}$$

$$A^n = P \begin{pmatrix} 1 & 0 \\ 0 & 2^n \end{pmatrix} P^{-1}$$

$$A^n = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 2^n \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$$

$$A^n = \begin{pmatrix} 1 & 2^n \\ 1 & 2^{n+1} \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$$

$$A^n = \begin{pmatrix} 2-2^n & -1+2^n \\ 2-2^{n+1} & -1+2^{n+1} \end{pmatrix}$$

$$3) \begin{pmatrix} u_n \\ u_{n+1} \end{pmatrix} = U_n = A^n U_0 = \begin{pmatrix} 2-2^n & -1+2^n \\ 2-2^{n+1} & -1+2^{n+1} \end{pmatrix} \begin{pmatrix} u_0 \\ u_1 \end{pmatrix}$$

$$\text{d'où } u_n = (2-2^n)u_0 + (-1+2^n)u_1.$$